Subliminal Channel - Steganography
using ElGamal Signature

Bob was put in jail.

Subliminal Channel - Pasąmonės Kanalas
Paslėptas Kanalas

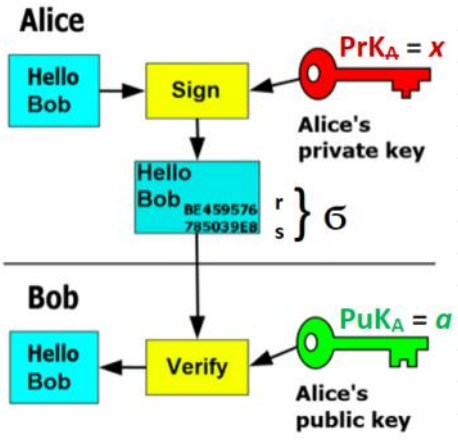
Grafity pictures

Repetition

>> p = 268 435 019; % 2^28-1 --> >> int64(2^28-1) % ans = **268 435 455**
>> g=2; % >> dec2bin(2^28-1)
% ans = 1111 1111 1111 1111 1111 1111 1111
>> p=268435019;
>> g=2;

ElGamal Signature: Sign(PrK,h)=σ=(r,s)

A: B:



>> z=int64(randi(p-1))
z = 100497451 → z = 100497451
>> c=mod_exp(g,z,p)
c = 91968695 → c = 91968695

Let **M** is a message to be signed which can be arbitrary chosen.
To sign **M** the **h**-value of **M** must be computed: **h=H(M)**.
The secret message to be sent is a number **k** satisfying the following conditions:
1. Number **k<p**.
2. gcd(**k**, **p**-1)=1.

1.***Signature generation on* h:** Alice generates secret number **k<p**, **gcd(k, p — 1) = 1**, **k** and **p-1** are relatively prime.
The secret message Alice intends to send to Bob is encoded by **k**.

- Compute **k^{-1} mod (p-1)**: **k^{-1} mod** (**p — 1**) exists if **gcd(k, p — 1) = 1**, **k** and **p-1** are relatively prime.
k^{-1} can be found using either Extended Euclidean algorithmt or Euler theorem

>> k_m1=mulinv(k,p-1)	**%** **k^{-1}**mod (**p**-1) computation.

- Compute **$r=g^k$ mod p**
- Compute **s=(h-z*r)*k^{-1} mod (p-1)** --> **h=z*r+s*k mod (p-1)**.

Signature **σ=(r,s)**
For subliminal channel creation s^{-1} mod (p-1) must exists, such that **$s*s^{-1}$= 1 mod (p-1)**: gcd(s,p-1)=1.
>> s_m1=mulinv(s,p-1) **%** **s^{-1}**mod (**p**-1) computation.

```
>>  p =   int64(268435019)
p = 268435019
>> g=2;
>> z=int64(randi(p-1))
z = 7688965
>> c=mod_exp(g,z,p)
c = 217126641
```

```
>> M='Bob I wait you'
M = Bob I wait you
>> h=hd28(M)
h = 240745341
```

```
>> s=int64(126312337)
s = 126312337
>> gcd(s,p-1)
ans = 1
```

```
>> s_m1=int64(212931827)
s_m1 = 212931827
>> mod(s*s_m1,p-1)
ans = 1
```

$gcd(s, p-1) = 1$

A: $M, \sigma = (r, s)$ → (c) → B: $h = H(M)$

2.**Signature on h verification**

A signature (r,**s**) on message **h** is verified as follows.

1. 1<**r**<**p-1** and 1<**s**<**p-1.**
2. **V1=g^h mod p, V2=c^r r^s mod p,** and **V1=V2.**

The verifier accepts a signature if all conditions are satisfied and rejects it otherwise.

2.**Secret message h=k recovery from the equation**

s=(h-z*r)*k^-1 mod (p-1) /*k

s*k=(h-z*r)*k^-1 *k mod (p-1)

s*k=(h-z*r) mod (p-1) /*s^-1

s must satisfy the condition that s^-1 mod (p-1) exists, such that **s*s^-1= 1 mod (p-1).**

s^-1*s*k=(h-z*r)* mod (p-1)

k=(h-z*r)* mod (p-1)

```
>> s_m1=mulinv(s,p-1)

>> k=mod(h-z*r,p-1)
```

```
k=int64(206415161)
gcd(k,p-1)
ans=1
r=mod_exp(g,k,p)
r = 113543563
```

Subliminal Channel - Steganography
Using Schnorr Signature

M - masking message to be signed.

$h' = k$ - secret message to be sent.

$r = g^k \bmod p$

$h = H(M \| r)$

$s = k + z * h \bmod (p-1)$

$M, \sigma = (r, s)$ → B: $h = H(M \| r)$

$V1 = g^s \bmod p$,

gcd(**z**, **p**-1) = 1, then **z** $^{-1}$ mod(**p**-1) exists.

$s = k + z * h \mod (p-1)$

$V1 = g^s \mod p$

$V2 = r * c^h \mod p$

$V1 \stackrel{?}{=} V2$

$k = (s - z * h) \mod (p-1) = n'$

>> p = int64(268435019)
p = 268435019
>> g=2;
>> z=int64(randi(p-1))
z = 7688965
>> c=mod_exp(g,z,p)
c = 217126641

>> M='Bob I wait you impatiently'
M = Bob I wait you
>> h=hd28(M)
h = 240745341

>> k=int64(randi(p-1))
k = 21398217
>> r=mod_exp(g,k,p)
r = 92029919
>> s=mod(k+z*h,p-1)
s = 34241676

>> V1=mod_exp(g,s,p)
V1 = 63032245
>> c_h=mod_exp(c,h,p)
c_h = 62405921
>> V2=mod(r*c_h,p)
V2 = 63032245

>> kk=mod(s-z*h,p-1)
kk = **21398217**

Bit Commitment using H-function

B : Should I must sell my bitcoins ?
A : Don't hurry, I know the price for the next month.
B : Then tell me please,
A : I'll tell you next month, but if you want to know immediatly give me 3 BTC.
B : How it is to know me that you are not cheating?
A : We can use Bit Commitment scheme,

Bit Commitment using MAC

BTC-NM
$h = H(BTC\text{-}NM \| rand)$

$h \rightarrow$ B:
After 1 month
1 BTC = 27 000 $
$\leftarrow$ Send me your guess
$BTC\text{-}NM \| rand \rightarrow$
$h' = H(BTC\text{-}NM \| rand)$
$h \stackrel{?}{\neq} h'$

Bit Commitment using HMAC

A: $k = lsb_{256}(z)$ symmetric key

B: $k = lsb_{256}(z)$

$HMAC(k, BTC\text{-}NM \| rand) = h$ —h→ after 1 month

Send me your guess

$Enc(k, BTC\text{-}NM \| rand) = c$ —c→ $Dec(k, c) = BTC\text{-}NM \| rand$

$h' = HMAC(k, BTC\text{-}NM \| rand)$

If $h = h' \Rightarrow$ A guessed BTC-NM.

Bit Commitment using RSA (partially)

RSA key generation: 2048 bits arithmetics; we will use 28 bits arithm.

1. Two primes p, q are generated at random. $|q| = 1024$ bits $|p| = 1024$ bits
2. RSA module $n = p \cdot q$ is computed & $\phi(n) = (p-1)\cdot(q-1) = \phi$.
3. Random RSA exponent e: $\gcd(e, \phi) = 1$ is computed.
   According to RSA standard $e = 2^{16}+1$.
4. The inverse element to e mod ϕ is computed:
   $d = e^{-1} \bmod \phi \Rightarrow d\,e \bmod \phi = 1$.
5. $PrK = d$; $PuK = (n, e)$.

```
>> e=2^16+1
e = 65537
>> isprime(e)
ans = 1
```

We use $|n| = 28$ bits → $|p| = |q| = 14$ bits

RSA textbook encryption

m - message: $m < n \sim 2^{2048}$; $|m| < 2048$ bits

$m \bmod n = m$

B: PuK_A ← A: $PuK_A = (n, e)$; $PrK_A = d$.

$c = Enc(PuK_A, m) = m^e \bmod n$

```
>> m=int64(111222333)
m = 111222333
>> c=mod_exp(m,e,n)
c = 51722206
```

c →

$Dec(PrK_A, c) = m =$

$= c^d \bmod n =$

$= (m^e)^d = m^{ed} =$

$= m^{ed \bmod \phi} \bmod n =$ ($ed \bmod \phi = 1$)

```
>> mm=mod_exp(c,d,n)
mm = 111222333
```

```
>> c=mod_exp(m,e,n)
c = 51722206
```

$= m^{ed \bmod \phi} \bmod n = m^{1} \bmod n = m \bmod n = m$ ($ed \bmod \phi = 1$)

RSA textbook encryption is not randomised – is not probabilistic.

Key Generation

Public Parameter $PP = (p)$ p - may be strong prime

A: $K_A = (e_A, d_A)$ $e_A \cdot d_A = 1 \bmod (p-1)$

B: $K_B = (e_B, d_B)$ $e_B \cdot d_B = 1 \bmod (p-1)$ % private keys

$e_A \leftarrow randi : gcd(e_A, p-1) = 1 \Rightarrow \exists! \; d_A = e_A^{-1} \bmod (p-1)$

$d_A = e_A^{-1} \bmod p$

```
>> p
p = 268435019
>> eA=int64(randi(p-1))
eA = 113108923
>> gcd(eA,p-1)
ans = 1
>> dA=mulinv(eA,p-1)
dA = 114348687
>> mod(eA*dA,p-1)
ans = 1
```

$d_B = e_B^{-1} \bmod p$

```
>> eB=int64(randi(p-1))
eB = 175845008
>> gcd(eB,p-1)
ans = 2
>> eB=int64(randi(p-1))
eB = 213299919
>> gcd(eB,p-1)
ans = 1
>> dB=mulinv(eB,p-1)
dB = 182812595
>> mod(eB*dB,p-1)
ans = 1
```

M – message : Bitcoin price next month

A: Encrypts M with encryption function E and sends ciphertext c_1 to B. $M < p$.

$Enc(e_A, M) = M^{e_A} \bmod p = c_1$

$\xrightarrow{c_1}$ B: $Enc(e_B, c_1) = c_1^{e_B} \bmod p = c_2$

$\xleftarrow{c_2}$

After 1 month

$A: c_3 = c_2^{d_A} \bmod p \xrightarrow{c_3} B: c_4 = c_3^{d_B} = \dots$

$$= \left(c_2^{d_A}\right)^{d_B} = \left(\left(c_1^{e_B}\right)^{d_A}\right)^{d_B} =$$

$$= \left(\left(\left(M^{e_A}\right)^{e_B}\right)^{d_A}\right)^{d_B} \bmod p =$$

$$= M^{(e_A d_A)(e_B d_B) \bmod (p-1)} \bmod p =$$

$e_A d_A \bmod (p-1) = 1$ & $e_B d_B \bmod (p-1) = 1$

$$= M^{1 \cdot 1} \bmod p \overset{M<p}{=\!=} M$$